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Procedia Engineering 127 (2015) 1278 – 1284

Procedia Engineering

www.elsevier.com/locate/procedia

International Conference on Computational Heat and Mass Transfer-2015

On Series Solution of MHD Flow over a Stretching Permeable Surface

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Abstract

The objective of present paper is to revisit the problem pertaining to the boundary layer equations governing the flow of viscous fluid due to a stretching surface. The series solution method has been effectively implemented to a related integro-differential equation. The condition at infinity is applied to a related Pade approximants of the obtained series solution. The features of the flow characteristic have been analyzed and discussed. Comparison of the obtained results for some particular cases of the present study has been done with the earlier results and they have been found in good agreement.

Peer-review under responsibility of the organizing committee of ICCHMT – 2015

Keywords: Similarity boundary layer; Series Solution; Pade Approximants; Heat and Mass Transfer.

Nomenclature

α_0	constant
η	similarity variable
ν	kinematic viscosity
ρ	fluid density
σ	electrical conductivity
B_0	magnetic field
f	dimensionless stream function
K	suction/injection parameter
M	magnetic parameter
u	velocity along x-axis
v	velocity along y- axis
V_0	suction/injection velocity

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Peer-review under responsibility of the organizing committee of ICCHMT – 2015
doi:10.1016/j.proeng.2015.11.484

1. Introduction

The flow of a viscous fluid on account of a stretching surface is of practical significance in the fields of metallurgy and chemical engineering, and hence has been extensively studied by [1]. The magneto-hydrodynamic (MHD) flow problems have recently become more important from industrial view-point, especially from metallurgical view-point, where-in the purification of the molten metal from non-metallic inclusions by the application of magnetic field is being done. The first attempt to study the MHD flow over a stretching wall in an electrically conducting fluid was taken by [2]. Further contributions to problems of this type were given by [3], [4], [5], [6], [7], [8,9], [10], [11], etc. However, the effect of the mass transfer pertaining to the similarity solution for the MHD flow over a stretching permeable surface subject to suction or injection has received only a limited attention so far. The contributions made by authors [3], [4], [7] [12], [13], [14], [15], [16], [17] and [18] are worth while mentioning in this regard. These studies are, however, restricted only to relatively low values of mass transfer parameter, K (say). The extension of the problem to include the large values of K still remains incomplete. However, [19] have tried to tackle this kind of problem to some extent by taking into account the large values of the mass transfer parameters.

The objective of the present paper is to obtain the solution of the third order non-linear differential equation pertaining to the boundary layer equations governing the flow of viscous fluid due to a stretching surface by using series solution as suggested by [20]. This problem has already been tackled by [19] by using perturbation technique for large values of the suction and injection parameters. In our attempt, the condition at infinity has been applied to a related Pade approximants of the obtained solutions. The features of the flow field have been analysed and the results obtained in this paper have been compared graphically with those of [19]. Our results have been found in excellent agreement with those of [19].

2. Basic Equations

Let us consider the flow of an electrically conducting incompressible fluid (with electrical conductivity σ) over a permeable wall coinciding with the plane $y = 0$; the flow being confined to $y > 0$. Two equal and opposite forces are introduced along the $x-$ axis so that the wall is stretched keeping the origin fixed, and a uniform magnetic field B_0 is imposed along the $y-$ axis. The basic boundary layer equations for the stretching flow are [21]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

where u and v are the velocity components along $x-$ and $y-$ axes, and ρ and ν are the density and kinematic viscosity of the fluid, respectively. The appropriate boundary conditions to the problem are

$$u = ax, \quad v = -V_0 \;\; at \;\; y = 0; \quad u \to 0 \;\; at \;\; y \to 0 \tag{3}$$

where $a > 0$, and V_0 is the velocity of suction ($V_0 > 0$) or injection ($V_0 < 0$), respectively.

Defining the variables

$$u = axf'(x), \quad v = -(a\nu)^{1/2} f(\eta), \quad \eta = \left(\frac{a}{\nu}\right)^{1/2} y \tag{4}$$

and substituting them into equation (2), we get the following Falkner -Skan type [22] ordinary differential equation

$$f''' + ff'' - f'^2 - Mf' = 0 \tag{5}$$

with the boundary conditions

$$f(0) = K, \qquad f'(0) = 1, \qquad f'(\infty) = 0 \tag{6}$$

where the prime denotes differentiation w. r. t. η. Here $M = \frac{\sigma B_0^2}{a\rho}$ is the magnetic parameter and $K = \frac{V_0}{(a\nu)^{1/2}}$ is the suction ($K > 0$) or injection ($K < 0$) parameter [19].

Pade approximant is the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of a function (Reference [23], [20] and [24]). The $[\mathcal{L}/\mathcal{M}]$ Pade approximant to a formal power series is given by $[\mathcal{L}/\mathcal{M}] = P_{\mathcal{L}}(x)/Q_{\mathcal{M}}(x)$, where $P_{\mathcal{L}}(x)$ is a polynomial of degree at most $\mathcal{L}$ and $Q_{\mathcal{M}}(x)$ is a polynomial of degree at most $\mathcal{M}$. Without loss of generality, we can assume $Q_{\mathcal{M}}(0)$ to be 1. Furthermore, $P_{\mathcal{L}}(x)$ and $Q_{\mathcal{M}}(x)$ have no common factors. This means that the formal power series $A(x)$ equals the $[\mathcal{L}/\mathcal{M}]$ approximant through $\mathcal{L} + \mathcal{M} + 1$ terms. It is a well known fact that Pade approximants will converge on the entire real axis if $f(\eta)$ is free of singularities on the entire real axis. Other important contribution on Pade approximants are due to [25], [26] and [27].

3. Analysis

Here we consider the initial value problem (5) with the boundary conditions

$$f(0) = K, \qquad f'(0) = 1, \qquad f''(0) = \alpha_0(say), \tag{7}$$

and

$$f'(\infty) = 0 \tag{8}$$

The equation (5) can be written as

$$f''' = -ff'' + f'^2 + Mf' \tag{9}$$

Integrating both the sides of (9) from 0 to η and using the condition $f''(0) = \alpha_0$, yields

$$f''(\eta) = (\alpha_0 - MK) - \int_0^\eta f(t)f''(t)dt + \int_0^\eta (f'(t))^2 dt + Mf(t) \tag{10}$$

Integrating both sides of (10) from 0 to η and noting the condition $f'(0) = 1$, yields

$$f'(\eta) = 1 + (\alpha_0 - MK)\eta - \int_0^\eta \int_0^\eta f(t)f''(t)dtdt + \int_0^\eta \int_0^\eta f'(t)^2 dtdt + M\int_0^\eta f(t)dt \tag{11}$$

so that by integrating again, we obtain

$$\begin{aligned} f(\eta) = K + \eta + \frac{(\alpha_0 - MK)\eta^2}{2} - \int_0^\eta \int_0^\eta \int_0^\eta f(t)f''(t)dtdtdt \\ + \int_0^\eta \int_0^\eta \int_0^\eta f'(t)^2\, dtdtdt + M\int_0^\eta \int_0^\eta f(t)dtdt \end{aligned} \tag{12}$$

The following integro-differential equation

$$\begin{aligned} f(\eta) = K + \eta + \frac{(\alpha_0 - MK)\eta^2}{2} - \frac{1}{2}\int_0^\eta (\eta - t)^2 f(t)f''(t)dt \\ + \frac{1}{2}\int_0^\eta (\eta - t)^2 (f'(t))^2 dt + M\int_0^\eta (\eta - t)f(t)dt \end{aligned} \tag{13}$$

is obtained from (12) upon converting the triple integral to a single integral.

To determine a solution of (13), we use the series solution method used by [20]. Therefore, we express $f(\eta)$ as a power series of the form

$$f(\eta) = \sum_{n=0}^{\infty} a_n \eta^n \tag{14}$$

Substituting (14) into both sides of (13), and by using only a few terms for simplicity reasons, we find

$$\begin{aligned}
&a_0 + \eta a_1 + \eta^2 a_2 + \eta^3 a_3 + ...\\
&= K + \eta + \frac{(\alpha_0 - MK)\eta^2}{2} - \int_0^\eta \left(\frac{1}{2}\eta^2 - \eta t + \frac{1}{2}t^2\right)(a_0 + ta_1\\
&+t^2 a_2 + t^3 a_3 + t^4 a_4 + t^5 a_5 + t^6 a_6 + t^7 a_7 + t^8 a_8 + t^9 a_9 + t^{10} a_{10} + ...)\\
&\times\left(2a_2 + 6ta_3 + 12t^2 a_4 + 20t^3 a_5 + 30t^4 a_6 + 42t^5 a_7 + 56t^6 a_8\right.\\
&\left.+72t^7 a_9 + 90t^8 a_{10} + ...\right) dt + \int_0^\eta \left(\frac{1}{2}\eta^2 - \eta t + \frac{1}{2}t^2\right)(a_1 + 2ta_2\\
&+3t^2 a_3 + 4t^3 a_4 + 5t^4 a_5 + 6t^5 a_6 + 7t^6 a_7 + 8t^7 a_8 + 9t^8 a_9\\
&+10t^9 a_{10} + ...)^2 dt + \int_0^\eta (\eta M - Mt)\left(a_0 + ta_1 + t^2 a_2 + t^3 a_3\right.\\
&\left.+t^4 a_4 + t^5 a_5 + t^6 a_6 + t^7 a_7 + t^8 a_8 + t^9 a_9 + t^{10} a_{10} + ...\right) dt
\end{aligned} \tag{15}$$

which gives after integration and simplification

$$\begin{aligned}
&a_0 + \eta a_1 + \eta^2 a_2 + \eta^3 a_3 + \eta^4 a_4 + \eta^5 a_5 + \eta^6 a_6 + \eta^7 a_7 + ...\\
&= K + \eta + \frac{1}{2}\eta^2(-KM + \alpha_0) + \frac{1}{2}M\eta^2 a_0 + \frac{1}{6}M\eta^3 a_1 + \frac{1}{6}\eta^3 a_1^2 + ...
\end{aligned} \tag{16}$$

Equating the coefficients of like powers of η in both sides leads to

$$\begin{aligned}
&a_0 = K, \quad a_1 = 1, \quad a_2 = \frac{\alpha_0}{2}, \quad a_3 = \frac{1}{6}(1 + M - K\alpha_0),\\
&a_4 = \frac{1}{24}\left(-K - KM + \alpha_0 + K^2\alpha_0 + M\alpha_0\right),\\
&a_5 = \frac{1}{120}\left(K^2 + M + K^2 M + M^2 - K\alpha_0 - K^3\alpha_0 - 2KM\alpha_0 + \alpha_0^2\right),\\
&a_6 = \frac{1}{720}\left(K - K^3 - KM - K^3 M - 2KM^2 + \alpha_0 + K^4\alpha_0\right.\\
&\left.+2M\alpha_0 + 3K^2 M\alpha_0 + M^2\alpha_0 - 3K\alpha_0^2\right), ...
\end{aligned} \tag{17}$$

Accordingly, the solution of the equation (5) in a series form is given by

$$\begin{aligned}
&f(\eta) = K + \eta + \frac{\eta^2\alpha_0}{2} + \frac{\eta^3}{6}(1 + M - K\alpha_0)\\
&+\frac{\eta^4}{24}\left(-K - KM + \alpha_0 + K^2\alpha_0 + M\alpha_0\right)\\
&+\frac{\eta^5}{120}\left(K^2 + M + K^2 M + M^2 - K\alpha_0 + K^3\alpha_0 + 2KM\alpha_0 + \alpha_0^2\right) + ...
\end{aligned} \tag{18}$$

$$\begin{aligned}
&f'(\eta) = 1 + \eta\alpha_0 + \frac{\eta^2}{2}(1 + M - K\alpha_0)\\
&+\frac{\eta^3}{6}\left(-K - KM + \alpha_0 + K^2\alpha_0 + M\alpha_0\right) + ...
\end{aligned} \tag{19}$$

4. Results and Discussion

We here note that the series solution (18) can also be obtained by using the series solution near the ordinary point $\eta = 0$ directly into the nonlinear equation (5). Further, the result (18) may also be determined by using the Adomian's decomposition method [28]. Our goal now is to determine the constant α_0 by using the condition $f'(\infty) = 0$. It is easily seen that this condition cannot be applied directly to (19). We can achieve our goal by representing the series (19) by a rational function $h(\eta)$ by using the powerful Pade approximants $[\mathcal{L}/\mathcal{M}]$ of this series. From [23], a Pade approximant to the power series (19) is defined as

$$[\mathcal{L}/\mathcal{M}] = \frac{P_{\mathcal{L}}(\eta)}{Q_{\mathcal{M}}(\eta)} \tag{20}$$

where $P_{\mathcal{L}}(\eta)$ and $Q_{\mathcal{M}}(\eta)$ are polynomials of degrees at most $\mathcal{L}$ and $\mathcal{M}$ respectively. Besides, we may consider $Q_{\mathcal{L}}(0) = 1$, and $P_{\mathcal{L}}(\eta)$ and $Q_{\mathcal{M}}(\eta)$ have no common factors.

In the following, we have determined the Pade approximants [2/2] of (19). The Pade approximants [3/3] and [4/4] can be determined in a parallel manner. To determine the Pade approximants [2/2] to $f'(\eta)$ of degree 4, it requires choosing A_0, A_1, A_2, B_1 and B_2 so that the coefficients of η^k for $k = 0, 1, 2, 3, 4$ are zero in the expression

$$\begin{aligned}&\Big(1 + \eta\alpha_0 + \frac{\eta^2}{2}(1 + M - K\alpha_0) + \frac{\eta^3}{6}(-K - KM + \alpha_0 + K^2\alpha_0 + M\alpha_0)\\ &+\frac{\eta^4}{24}(K^2 + M + K^2M + M^2 - K\alpha_0 + K^3\alpha_0 + 2KM\alpha_0 + \alpha_0^2) + ...\Big)\\ &\times\left(1 + B_1\eta + B_2\eta^2\right) = \left(A_0 + A_1\eta + A_2\eta^2\right)\end{aligned} \tag{21}$$

Expanding (21), by putting $M = K = 0$ and equating the coefficients of η^k for $k = 0, 1, 2, 3, 4$ to zero yields Pade [2/2].
Consequently, the Pade approximant [2/2] is given by

$$[2/2] = \frac{1 + \frac{\eta\left(-4\alpha_0+3\alpha_0^3\right)}{2\left(-3+2\alpha_0^2\right)} + \frac{\eta^2\left(-18+23\alpha_0^2-6\alpha_0^4\right)}{12\left(-3+2\alpha_0^2\right)}}{1 - \frac{\eta^2\alpha_0^2}{12\left(-3+2\alpha_0^2\right)} + \frac{\eta\left(2\alpha_0-\alpha_0^3\right)}{2\left(-3+2\alpha_0^2\right)}}. \tag{22}$$

Applying the condition $f'(\infty) = 0$ to (22) gives

$$\alpha_0 = f''(0) = -1.04687. \tag{23}$$

In a manner parallel to our above discussion, the approximants [3/3] and [4/4] are obtained and the values of the constant $\alpha_0 = f''(0)$ are found to be very close to that of (23). Again, the value of $\alpha_0 = f''(0)$ is calculated for different values of the pertinent parameters K (for both suction and injection)and $M = 0, 1, 2$, which are tabulated below in the Tables 1-2.

Table 1. Values of $\alpha_0 = f''(0)$ for different values of M and $K \geq 0$

K	**Pade Approx.** $[\mathcal{L}/\mathcal{M}]$	**M=0**	**M=1**	**M=2**
0	[2,2]	-1.04687	-1.47078	-1.79821
	[3,3]	-1.01959	-1.40334	-1.72313
	[4,4]	-1.00591	-1.41606	-1.73323
1	[2,2]	-1.45045	-2.10895	-2.26012
	[3,3]	-1.8262	-2.02984	-2.29881
	[4,4]	-1.61111	-2.00721	-2.3138
2	[2,2]	-2.51771	-2.77893	-3.55769
	[3,3]	-2.60118	-2.80557	-3.59912
	[4,4]	-2.63818	-2.8745	-3.61063

From the Table 1, it is obvious that the skin friction i.e. $-f''(0)$ increases along with increasing values of the mass suction parameter ($K > 0$) and magnetic parameter (M) for the different Pade approximants. From the Table 2, it is obvious that the skin friction i.e. $-f''(0)$ increases along with increasing values of magnetic parameter (M) but

Table 2. Values of α_0 for different values of M and $K < 0$

K	**Pade Approx.** $[\mathcal{L}/\mathcal{M}]$	**M=0**	**M=1**	**M=2**
-1	[2,2]	-0.62224	-1.02147	-2.10588
	[3,3]	-0.616429	-1.0012	-2.19975
	[4,4]	-0.617707	-1.00006	-2.2506
-3	[2,2]	-2.97153	-0.52217	-0.74121
	[3,3]	-3.00092	-0.53372	-0.75002
	[4,4]	-0.30278	-0.56155	-0.78665
-5	[2,2]	-0.18571	-0.34889	-0.53321
	[3,3]	-0.191773	-0.35168	-0.54003
	[4,4]	-0.1958	-0.37215	-0.54138

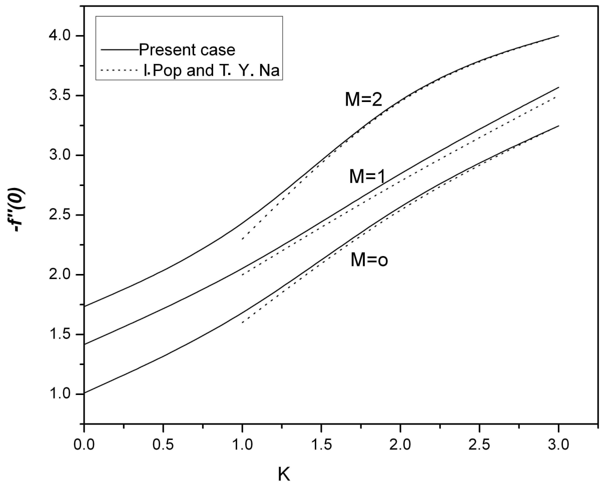

Fig. 1. Solution for large mass suction

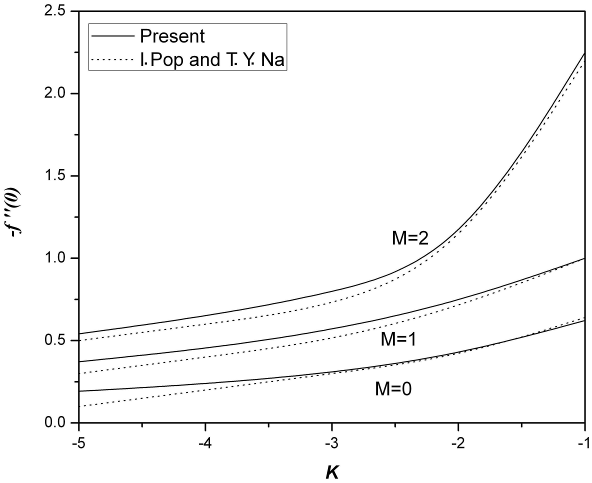

Fig. 2. Solution for large mass injection

decreasing values of the mass suction parameter ($K < 0$) for the different Pade approximants.

Thus these numerical results given in Tables 1 and 2 are highly in conformation with the graphical results given in Figures 1 and 2 respectively.

The effects of the mass suction parameter $K > 0$ and the magnetic parameter M have been depicted in the Figure 1. The present results have been compared with [19]. From the figures, it is evident that the present results are in good agreement with those obtained by [19].

The effects of the mass injection parameter $K < 0$ and the magnetic parameter M have been depicted in the Figure 2.The results in this case have also been compared and they are in good agreement with [19].

5. Concluding Remarks

In this paper, we have discussed the magnetohydrodynamic(MHD) flow of a viscous fluid due to a stretching surface. A series solution has been successfully obtained with the use of Pade approximants. The solutions are compared with the available results e.g. [19]. They have been found compatible with each other. From the figures, it is found that the skin friction increases with suction and the effect of injection is just opposite. On the other hand, the skin friction increases with M in the cases of suction and injection both.

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